

B.I.T. Sindri.

B.Tech. 1st Semester 2nd Mid semester Examination 2018.

Subject: Mathematics- I

Branch: All except C.Sc. & Engg

Time: 1.30 Hours

Full Marks : 20

Solve Five questions. Question no.1. is compulsory.

All questions are of equal value.

1. Answer any four questions. Choose the correct answer.

1X4

(a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and \vec{r} is a position vector of any point then curl \vec{r} is equal to

- (i) \vec{r} (ii) 3 (iii) 1 (iv) $\vec{0}$

(b) If $z = \frac{x^2+y^3}{x-y}$ then $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$ is equal to

- (i) 2z (ii) 0 (iii) 1 (iv) z.

(c) The magnitude of greatest rate of increase of $u = x^2 + yz^2$ at the point (1,-1,3) is equal to

- (i) 5 (ii) 11 (iii) 17 (iv) none of the above.

(d) If $f(x)$ is an odd function defined in the interval $[-\pi, \pi]$ then Fourier expansion of $f(x)$ is equal to

- (i) $\sum_{n=1}^{\infty} b_n \sin nx$ (ii) $\sum_{n=1}^{\infty} a_n \cos nx$ (iii) $\frac{a_0}{2}$ (iv) 0.

(e) If $\lim_{n \rightarrow \infty} u_n \neq 0$ then the series $\sum u_n$ is

- (i) convergent (ii) oscillatory (iii) divergent. (iv) none of the above.

(f) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ is

- (i) convergent (ii) divergent (iii) absolutely convergent (iv) none of the above.

(Turn over)

2. Test the convergence of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \dots, \infty, x > 0.$ 4

3. Examine the convergence of the series $\sum \frac{\sqrt{2n-1}}{(2n+2)(2n+4)}.$ 4

4. Expand $f(x) = x \sin x$ in a Fourier series in the interval $0 < x < 2\pi.$ 4

5. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ 4

6. In a plane triangle ABC find the maximum value of $\cos A \cdot \cos B \cdot \cos C.$ 4

7. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and \vec{r} is a position vector of any point and $r = |\vec{r}|$, show that $\text{div}(\text{grad } r^n) = n(n+1)r^{n-2}.$ 4

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